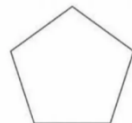
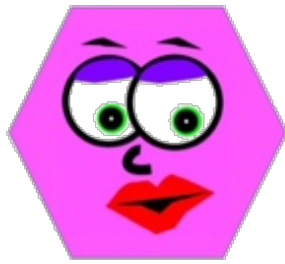


Polygons Questions By Topic:

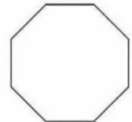


Pentagon



Hexagon

ME SOLVING  
FOR WHAT  
IS A POLYGON



Octagon



wagon

ME KNOWING  
EVERY  
COUNTRY'S FLAG



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Formulae Reminders

Let  $n$  = number of sides

Sum Of All Interior Angles



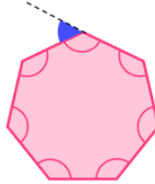
$$180(n - 2)$$

1 Interior Angle



$$\frac{180(n - 2)}{n}$$

1 Exterior Angle

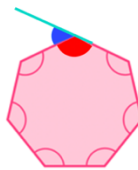


$$\frac{360}{n}$$

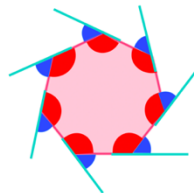
We can also use the formula

$$180 - \text{interior angle}$$

Why can we use the second formula? This is because the interior and exterior angles are **straight line angles**



$$\text{Interior} + \text{exterior} = 180^\circ$$



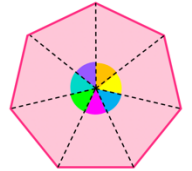
Number Of Sides

$$\frac{360}{\text{exterior angle}}$$

We can also use the formula

$$\frac{360}{180 - \text{interior angle}}$$

Angles At The Centre

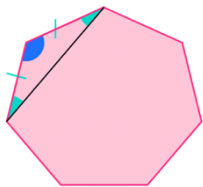


Each angle at the centre

$$\frac{360}{n}$$

You may also need to use some angle rules:

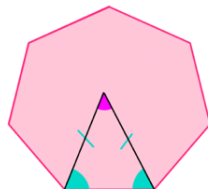
Isosceles Triangle



The base angles are equal

$$\text{each} = \frac{180 - \text{top angle}}{2}$$

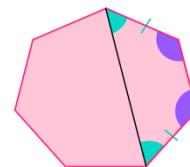
Isosceles Triangle



The base angles are equal

$$\text{each} = \frac{180 - \text{top angle}}{2}$$

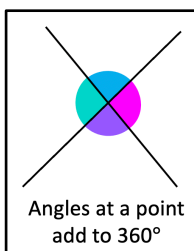
Isosceles Trapezoid



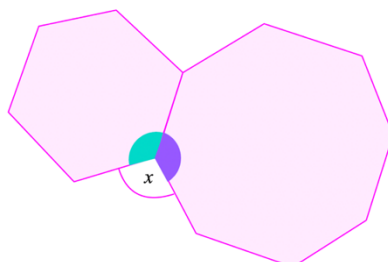
$$\text{each} = \frac{180 - \text{top angle}}{2}$$

$$\text{each} = \frac{180 - \text{bottom angle}}{2}$$

You may also need to deal with multiple polygons



Angles at a point add to  $360^\circ$



$$\frac{180(5 - 2)}{5} = 108^\circ$$

$$\frac{180(8 - 2)}{8} = 135^\circ$$

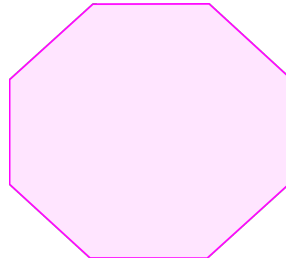
$$360 - 108 - 135 = 117^\circ$$

## 1 Bronze

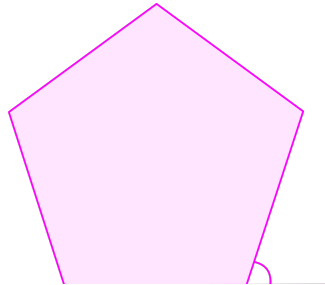


## 1.1 Working Out Angles

- 1) Work out the size of an exterior angle of a regular octagon.



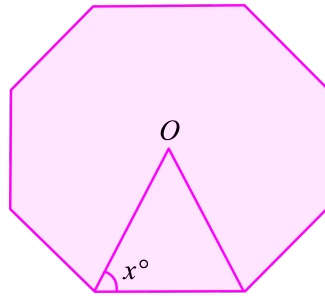
- 2) Work out the size of an exterior angle of a regular pentagon



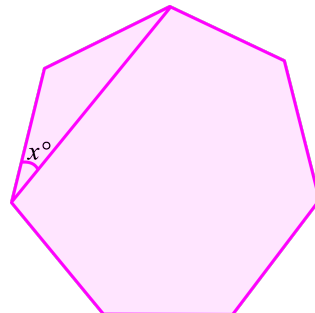
- 3) Find the sum of the interior angles of a polygon with 7 sides.
- 4) The diagram shows part of a regular 10-sided polygon. Work out the size of the angle marked  $x$ .



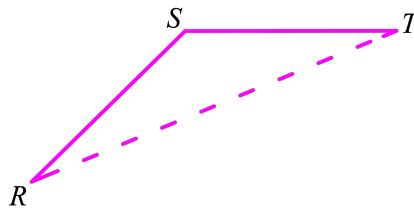
- 5) The diagram shows a regular octagon, with centre  $O$ . Work out the value of  $x^\circ$ .



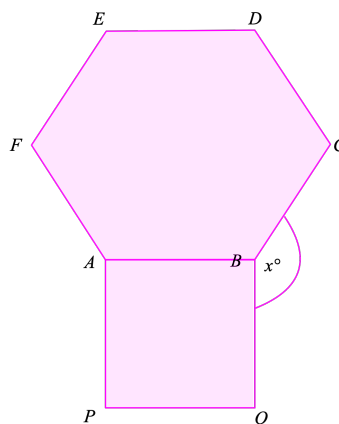
- 6) The diagram shows a regular polygon with 7 sides. Write out the value of  $x$ .



- 7)  $RS$  and  $ST$  are 2 sides of a regular 12-sided polygon.  $RT$  is a diagonal of a polygon. Work out the size of angle of  $STR$ .

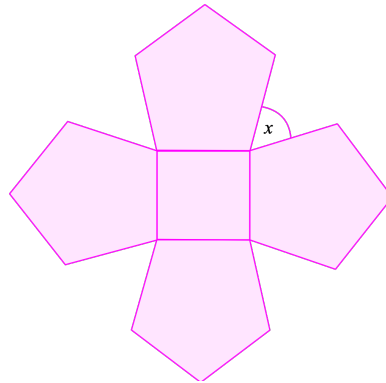


- 8)  $ABCDEF$  is a regular hexagon and  $ABQP$  is a square. Angle  $CBQ = x^\circ$ . Work out the value of  $x$

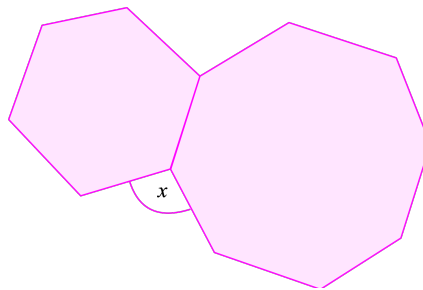




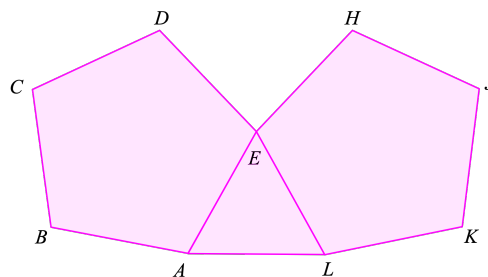
- 9) The diagram shows a square and 4 regular pentagons. Work out the size of the angle marked  $x$ .



- 10) The diagram shows a regular hexagon and a regular octagon. Calculate the size of the angle marked  $x$ . You must show all your working.



- 11) ABCDE and EHJKL are regular pentagons. AEL is an equilateral triangle. Work out the size of angle DEH.



## 1.2 Working Out The Number Of Sides

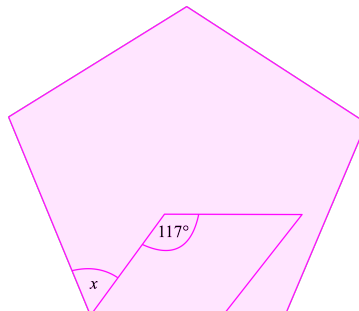
- 12) Each exterior angle of a regular polygon is  $30^\circ$ . Work out the number of sides of the polygon.
- 13) The size of each exterior angle of a regular polygon is  $18^\circ$
- Work out how many sides the polygon has
  - Work out the sum of the interior angles of the polygon
- 14) The size of each interior angle of a regular polygon is  $156^\circ$ . Work out the number of sides of the polygon

## 2 Silver

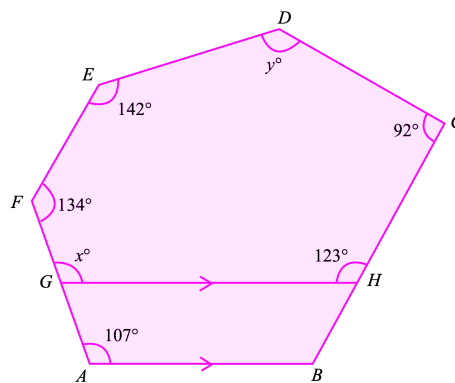


### 2.1 Working Out Angles

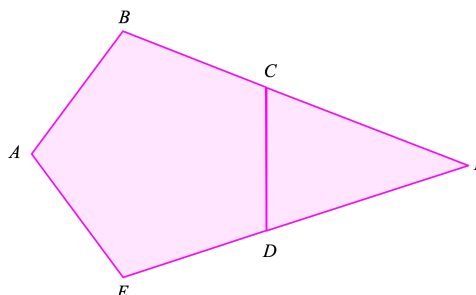
- 15) The diagram shows a regular pentagon and parallelogram. Work out the size of the angle marked  $x$ .



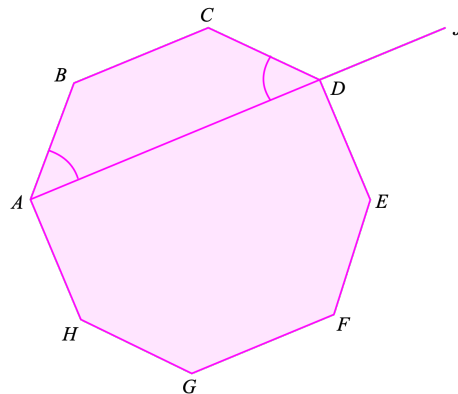
- 16) ABCDEF is a hexagon. G is a point on AF and H is a point on BC. GH is parallel to AB.
- Give a reason why  $x = 107$
  - Work out the value of  $y$



- 17) ABCDE is a regular pentagon. BCF and EDF are straight lines. Work out the size of angle CFD.

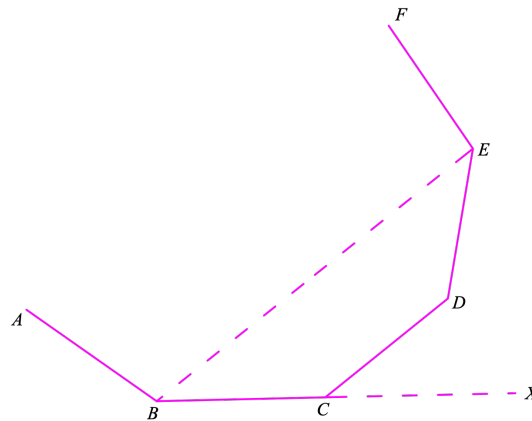


- 18)  $ABCDEFGH$  is a regular octagon.  $ADJ$  is a straight line.



angle  $BAD$  = angle  $CDA$   
 Show that angle  $CDJ = 135^\circ$

- 19)  $ABCDEF$  is part of a regular nonagon.  $BC$  is extended to  $X$ .  $B$  is joined to  $E$ . Calculate the size of



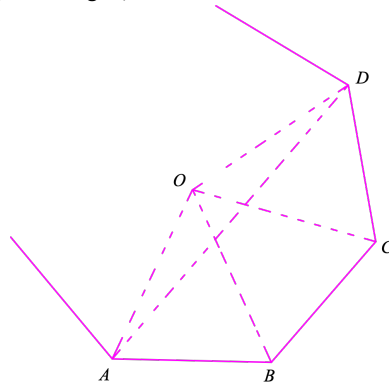
- i. Angle  $DCX$
- ii. Angle  $BCD$
- iii. Angle  $ABE$

### 3 Gold

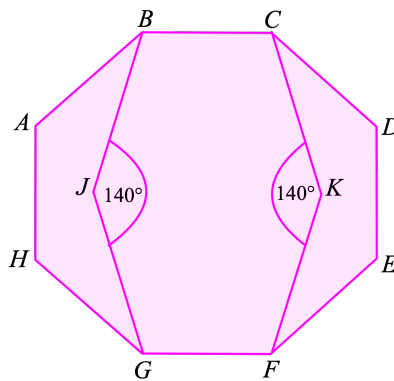


#### 3.1 Working Out Angles

20) ABCD forms three sides of a regular octagon, centre O. Calculate the size of angle BOC, OBC and OAD

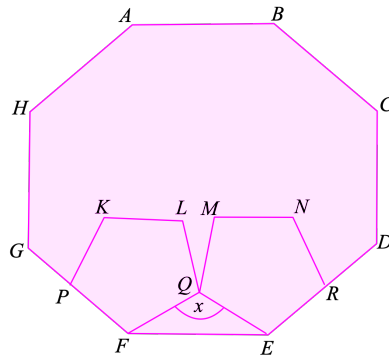


21) ABCDEFGH is a regular octagon

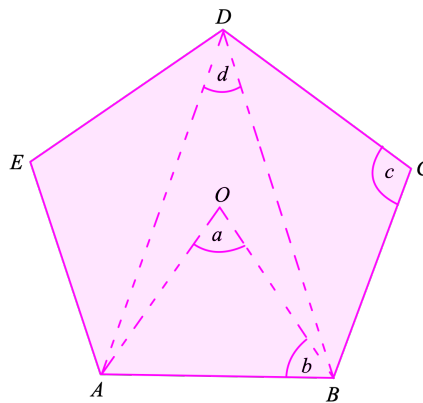


BCKFGJ is a hexagon  
 JK is a line of symmetry of the hexagon  
 Angle  $BJK = \text{angle } CKF = 140^\circ$   
 Work out the size of angle  $KFE$

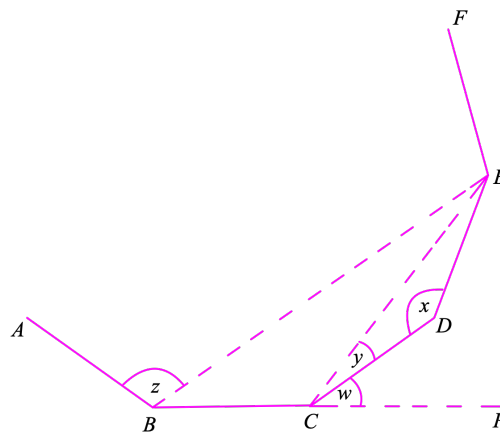
- 22) ABCDEFGH is a regular octagon. KLQFP and MNREQ are two identical regular pentagons. Work out the size of the angle marked  $x$



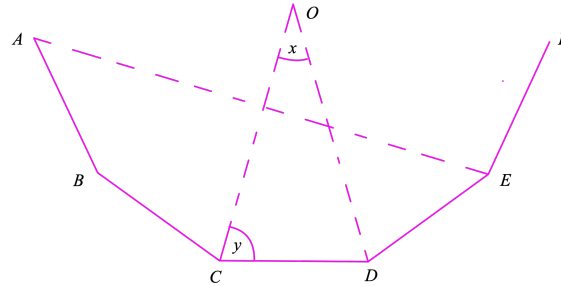
- 23) ABCDE is a regular polygon, centre O. Calculate the size of each of the angles marked a, b, c and d.



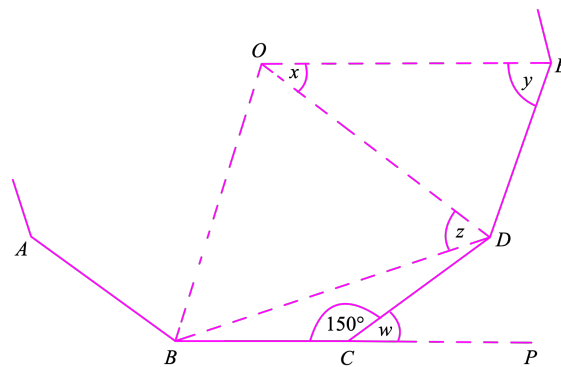
- 24) ABCDEF is part of a regular polygon with 10 sides. BCP is a straight line. Calculate the size of each of the angles marked  $w, x, y,$  and  $z$



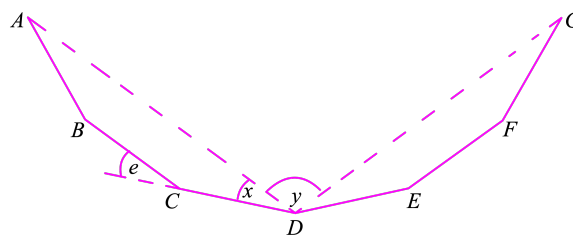
- 25) ABCDEF is part of a regular polygon, centre O. The size of angle COD and OCD are in the ratio 1:2. Calculate the size of angle
- COD
  - CDE
  - AED



- 26) A, B, C, D and E are corners of a regular polygon with centre O. BC is extended to P



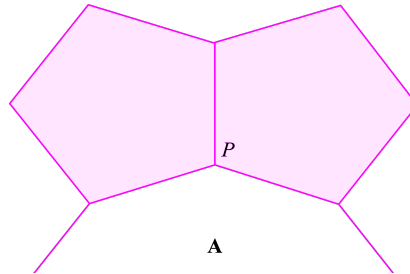
- Calculate the size the angle marked  $w$
  - How many sides does the polygon have
  - Calculate the size of each of the angles marked  $x, y,$  and  $z$
  - What type of triangle is OBD?
- 27) ABCDF is part of a regular 15-sided polygon. CD is extended to Z. Calculate



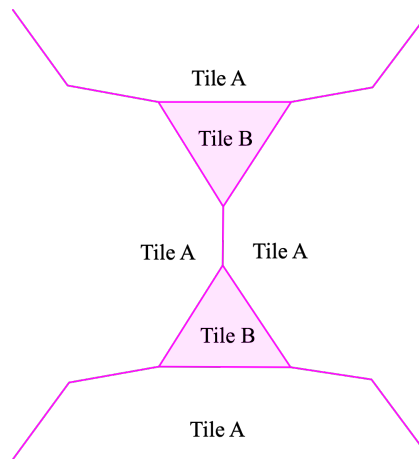
- the size of an exterior angle,  $e$
- the size of an interior angle
- the size of angle  $x$
- the size of angle  $y$

### 3.2 Working Out The Number Of Sides

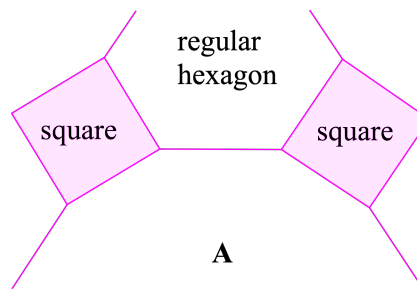
- 28) The diagram shows two congruent regular pentagons and part of a regular  $n$ -sided polygon **A**. Two sides of each of the regular pentagons and two sides of **A** meet at the point  $P$ . Calculate the value of  $n$ . show all your working clearly.



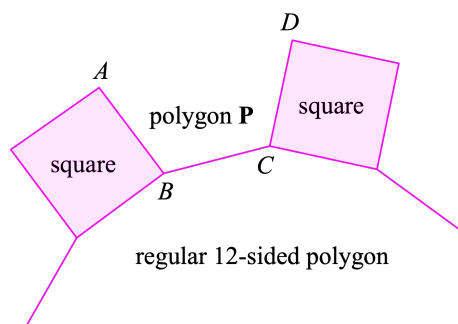
- 29) The diagram shows part of a pattern made from tiles. The pattern is made from two types of tiles, tile A and tile B. Both tile A and tile B are regular polygons. Work out the number of sides tile A has.



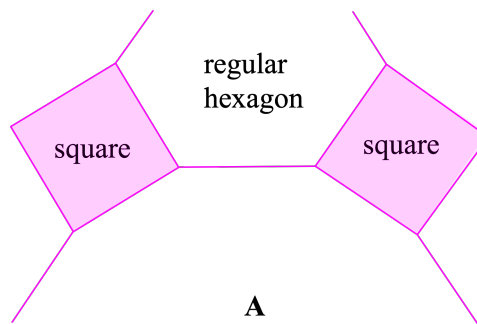
- 30) The diagram shows part of a tiling pattern. The tiling pattern is made from three shapes. Two of the shapes are squares and regular hexagons. The third shape is a regular  $n$ -sided polygon **A**. Work out the value of  $n$ .



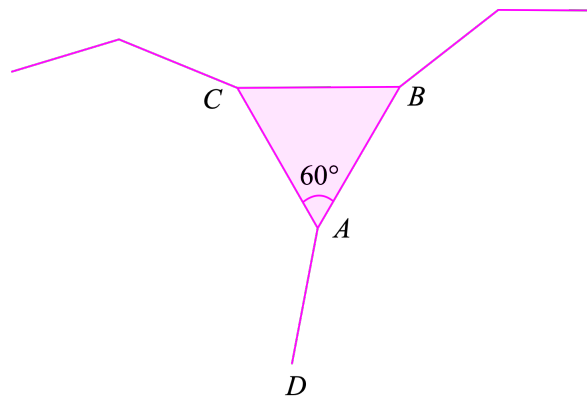
- 31)  $AB$ ,  $BC$  and  $CD$  are three sides of a regular polygon  $P$ . Show that polygon  $P$  is a hexagon. Show your working



- 32) The diagram shows part of a tiling pattern. The tiling pattern is made from three shapes. Two of the shapes are regular hexagons. The third shape is a regular  $n$ -sided polygon **A**. Work out the value of  $n$ .



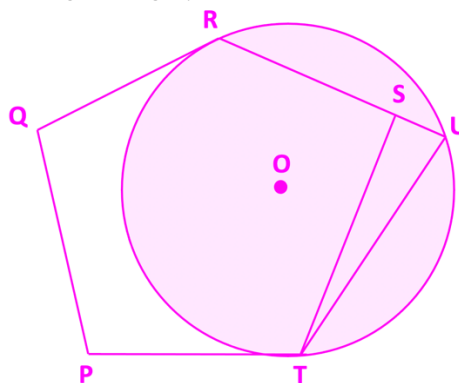
- 33) The sides of an equilateral triangle  $ABC$  and two regular polygons meet at the point  $A$ .  $AB$  and  $AD$  are adjacent sides of a regular 10-sided polygon.  $AC$  and  $AD$  are adjacent sides of a regular  $n$ -sided polygon. Work out the value of  $n$ .



- 34) A regular pentagon, a square and one other regular shape meet at a point and perfectly fit together leaving no gap. How many sides does this third mystery shape have and what is the sum of the interior angles?

### 3.3 With Circle Theorems

- 35)  $PQRST$  is a regular pentagon.  $R$ ,  $U$  and  $T$  are points on circle, centre  $O$ .  $QR$  and  $PT$  are tangents to the circle.  $RSU$  is a straight line. Prove that  $ST=UT$ .  
Hint: prove isosceles triangle by base angles being equal

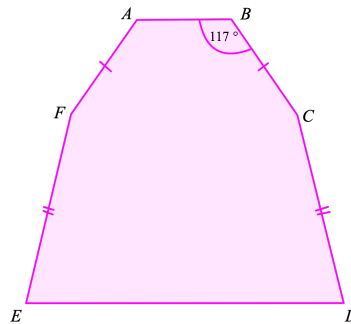




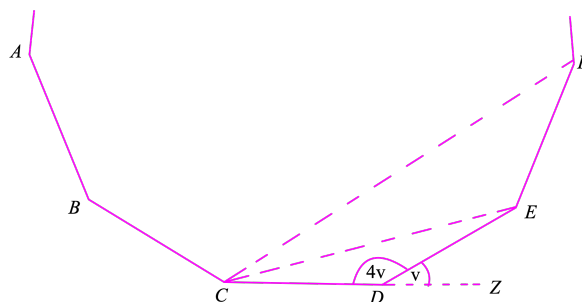
## 4 Diamond



- 36) The diagram shows a hexagon with 1 line of symmetry.  
 $FA = BE = CD$   
 Angle  $ABC = 117^\circ$   
 Angle  $BDC = 2 \times$  angle  $CDE$   
 Work out the size of angle  $AFE$ .



- 37) ABCDEF is part of a regular polygon. CD is extended to Z
- Calculate the size of the angle marked  $v$
  - Write down the number of sides of the regular polygon
  - Calculate the size of the angle  $DCE$
  - Calculate the size of the angle  $FEC$
  - Calculate the size of the angle  $EFC$



- 38) A Polygon has an interior angle exactly 6.5 times the size of an exterior angle. Determine if this shape could be a regular polygon.
- 39) An irregular polygon has 5 of its angles as  $79^\circ, 42^\circ, 49^\circ, 52^\circ$  and  $97^\circ$ . Explain why this shape cannot be a hexagon.

- 40) The diagram shows an incomplete regular polygon. The size of each interior angle is 140 degrees greater than the size of each exterior angle. Work out the number of the sides the regular polygon has.



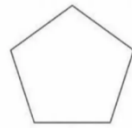
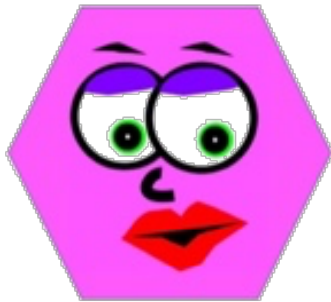
- 41) The diagram shows part of a regular polygon. The interior angle and the exterior angle at a vertex are marked. The size of the interior angle is 7 times the size of the exterior angle.



Work out the number of sides of the polygon.

- 42) The size of each interior angle of a regular polygon is 11 times the size of each exterior angle. Work out the number of sides the polygon has.

Polygons Questions By Topic Solutions

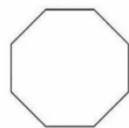


Pentagon



Hexagon

ME SOLVING  
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Octagon



wagon

ME KNOWING  
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## 1 Bronze



## 1.1 Working Out Angles

1)


<p><b>Way 1: Use formula for sum of interior angles <math>180(n - 2)</math></b></p> $\begin{aligned} \text{sum of all angles} &= 180(8 - 2) \\ &= 180(6) \\ &= 1080 \end{aligned}$ $1 \text{ interior angle} = \frac{1080}{8} = 135^\circ$ $\text{Exterior} = 180 - \text{interior} = 180 - 135 = 45^\circ$	<p><b>Way 2: Use formula for exterior angle <math>\frac{360}{n}</math></b></p> $\text{Exterior angle} = \frac{360}{8} = 45^\circ$

2)

<p><b>Way 1: Use formula for sum of interior angles <math>180(n - 2)</math></b></p> $\begin{aligned} \text{sum of all angles} &= 180(5 - 2) \\ &= 180(3) \\ &= 540 \end{aligned}$ $1 \text{ interior} = \frac{540}{5} = 108^\circ$ $\text{Exterior} = 180 - \text{interior} = 180 - 108 = 72^\circ$	<p><b>Way 2: Use formula for exterior angle <math>\frac{360}{n}</math></b></p> $\text{Exterior angle} = \frac{360}{5} = 72^\circ$

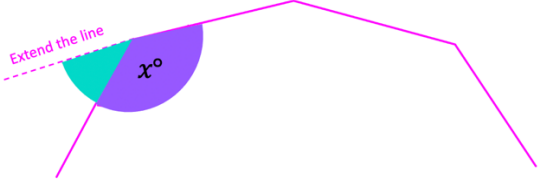
3)

Use formula for sum of interior angles  $180(n - 2)$



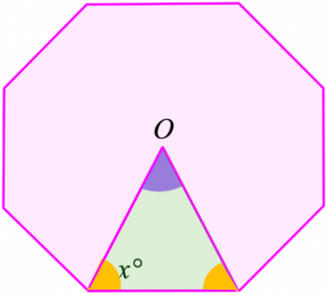
sum of all angles =  $180(7 - 2)$   
 $= 180(5)$   
 $= 900^\circ$

4)



<p>Way 1: Use formula for sum of interior angles <math>180(n - 2)</math></p> <p>sum of all angles = <math>180(10 - 2)</math>  <math>= 180(8)</math>  <math>= 1440</math></p> <p>1 interior = <math>x = \frac{1440}{10} = 144^\circ</math></p>	<p>Way 2: Use formula for exterior angle <math>\frac{360}{n}</math></p> <p>Exterior angle = <math>\frac{360}{10} = 36^\circ</math></p> <p>Interior angle = <math>x = 180^\circ - \text{exterior}</math>  <math>= 180 - 36 = 144^\circ</math></p>
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5)



Angle at the centre =  $\frac{360}{8} = 45^\circ$

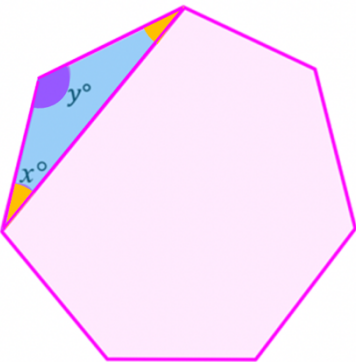
We have an isosceles triangle

Angles of a triangle add to  $180^\circ$

Base angles of an isosceles triangle are equal

$x = \frac{180 - 45}{2} = 67.5^\circ$

6)



<p><b>Way 1: Use formula for sum of interior angles <math>180(n - 2)</math></b></p> $\begin{aligned} \text{sum of all angles} &= 180(7 - 2) \\ &= 180(5) \\ &= 900 \end{aligned}$ $1 \text{ interior} = y = \frac{900}{7} = 128.571^\circ$	<p><b>Way 2: Use formula for exterior angle <math>\frac{360}{n}</math></b></p> $\text{Exterior angle} = \frac{360}{7} = 51.429^\circ$ $\text{Interior angle} = y = 180^\circ - \text{exterior}$ $= 180 - 51.429 = 128.571$
--	--

Now that we have the interior angle  $y$  we can work out  $x$  by looking at the triangle

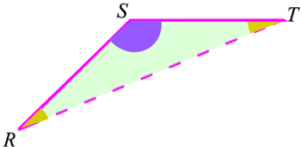
We have an isosceles triangle (since both sides of the triangle are the same length)

Angles of a triangle add to  $180^\circ$

Base angles of an isosceles triangle are equal

$$x = \frac{180 - 128.571}{2} = 25.715$$

7)



<p><b>Way 1: Use formula for sum of interior angles <math>180(n - 2)</math></b></p> $\begin{aligned} \text{sum of all angles} &= 180(12 - 2) \\ &= 180(10) \\ &= 1800 \end{aligned}$ $1 \text{ interior} = \angle RST = \frac{1800}{12} = 150^\circ$	<p><b>Way 2: Use formula for exterior angle <math>\frac{360}{n}</math></b></p> $\text{Exterior angle} = \frac{360}{12} = 30^\circ$ $\text{Interior angle} = \angle RST = 180^\circ - \text{exterior} = 180 - 30 = 150^\circ$
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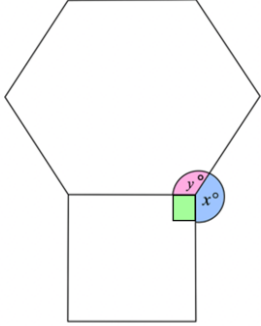
We have an isosceles triangle RST

Angles of a triangle add to  $180^\circ$

Base angles of an isosceles triangle are equal

$$\angle STR = \frac{180 - 150}{2} = 15^\circ$$

8)



Firstly, we consider the hexagon and find the interior angle  $y$

<p><b>Way 1: Use formula for sum of interior angles <math>180(n - 2)</math></b></p> <p>sum of all angles = <math>180(6 - 2)</math>  <math>= 180(4)</math>  <math>= 720</math></p> <p>● 1 interior = <math>y = \frac{720}{6} = 120^\circ</math></p>	<p><b>Way 2: Use formula for exterior angle <math>\frac{360}{n}</math></b></p> <p>Exterior angle = <math>\frac{360}{6} = 60^\circ</math></p> <p>● Interior angle <math>y = 180^\circ - \text{exterior}</math>  <math>= 180 - 60 = 120^\circ</math></p>
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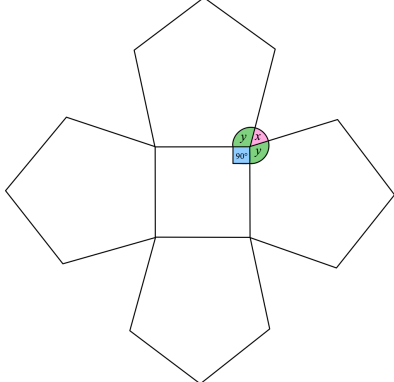
We have a pentagons connected to a square

ABPQ is a square so angle ABQ =  $90^\circ$

We also know that angles at a point add to  $360^\circ$

$x = 360 - 90 - 120 = 150^\circ$

9)



Firstly, we consider the pentagon and find the interior angle  $y$

<p><b>Way 1: Use formula for sum of interior angles <math>180(n - 2)</math></b></p> <p>sum of all angles = <math>180(5 - 2)</math>  <math>= 180(3)</math>  <math>= 540</math></p> <p>● 1 interior = <math>y = \frac{540}{5} = 108^\circ</math></p>	<p><b>Way 2: Use formula for exterior angle <math>\frac{360}{n}</math></b></p> <p>Exterior angle <math>x = \frac{360}{5} = 72^\circ</math></p> <p>● Interior angle = <math>180^\circ - \text{exterior} = 180 - 72 = 108^\circ</math></p>
--	--

We have two pentagons connected to a square

Each interior angle of a square inside is  $90^\circ$

We also know that angles at a point add to  $360^\circ$

●  $x = 360 - 108 - 108 - 90 = 54^\circ$

10)

We consider the hexagon and the octagon to find the interior angles  $y$  and  $z$

Way 1: Use formula for sum of interior angles $180(n - 2)$	Way 2: Use formula for exterior angle $\frac{360}{n}$
<p><u>Hexagon:</u>      sum of all angles = <math>180(6 - 2)</math>  <math>= 180(4)</math>  <math>= 720</math></p> <p>● 1 interior = <math>y = \frac{720}{6} = 120^\circ</math></p> <p><u>Octagon:</u>      sum of all angles = <math>180(8 - 2)</math>  <math>= 180(6)</math>  <math>= 1080</math></p> <p>● 1 interior = <math>z = \frac{1080}{8} = 135^\circ</math></p>	<p><u>Hexagon:</u>      Exterior angle = <math>\frac{360}{6} = 60^\circ</math></p> <p>● 1 interior = <math>y = 180^\circ - \text{exterior}</math>  <math>= 180 - 60 = 120^\circ</math></p> <p><u>Octagon:</u>      Exterior angle = <math>\frac{360}{8} = 45^\circ</math></p> <p>● 1 interior = <math>z = 180^\circ - \text{exterior}</math>  <math>= 180 - 45 = 135^\circ</math></p>

We have an octagon and a hexagon connected together.

We also know that angles at a point add to  $360^\circ$

●  $x = 360 - 120 - 135 = 105^\circ$

11)

Consider the Pentagon and the triangle

We consider the pentagon and the triangle to find the interior angles  $y$  and  $z$

Way 1: Use formula for sum of interior angles $180(n - 2)$	Way 2: Use formula for exterior angle $\frac{360}{n}$
<p><u>Pentagon:</u>      sum of all angles = <math>180(5 - 2)</math>  <math>= 180(3)</math>  <math>= 540</math></p> <p>● 1 interior = <math>y = \frac{540}{5} = 108^\circ</math></p> <p><u>Triangle:</u>      sum of all angles = <math>180(3 - 2)</math>  <math>= 180(1)</math>  <math>= 180</math></p> <p>● 1 interior = <math>z = \frac{180}{3} = 60^\circ</math></p>	<p><u>Pentagon:</u>      Exterior angle = <math>\frac{360}{5} = 72^\circ</math></p> <p>● Interior angle = <math>y = 180^\circ - \text{exterior} = 180 - 72 = 108^\circ</math></p> <p><u>Triangle:</u>      Exterior angle = <math>\frac{360}{3} = 120^\circ</math></p> <p>● Interior angle <math>z</math>: <math>180^\circ - \text{exterior} = 180 - 120 = 60^\circ</math></p>

We have two pentagons and an equilateral triangle connected together.



We also know that angles at a point add to  $360^\circ$

$$\bullet x = 360 - 108 - 108 - 60 = 84^\circ$$

## 1.2 Working Out The Number of Sides

12) .

<p><b>Way 1: Use formula for sum of interior angles <math>180(n - 2)</math></b></p> <p>Interior angle = <math>180 - \text{exterior angle}</math></p> <p>Interior angle = <math>180 - 30 = 150^\circ</math></p> <p>1 interior: <math>\frac{180(n-2)}{n} = 150</math></p> <p>Solve for <math>n</math>:</p> $\frac{180n - 360}{n} = 150$ $180n - 360 = 150n$ $30n = 360$ $n = \frac{360}{30} = 12$	<p><b>Way 2: Use formula for exterior angle <math>\frac{360}{n}</math></b></p> <p>Exterior angle = <math>\frac{360}{30} = 12</math></p>
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13)

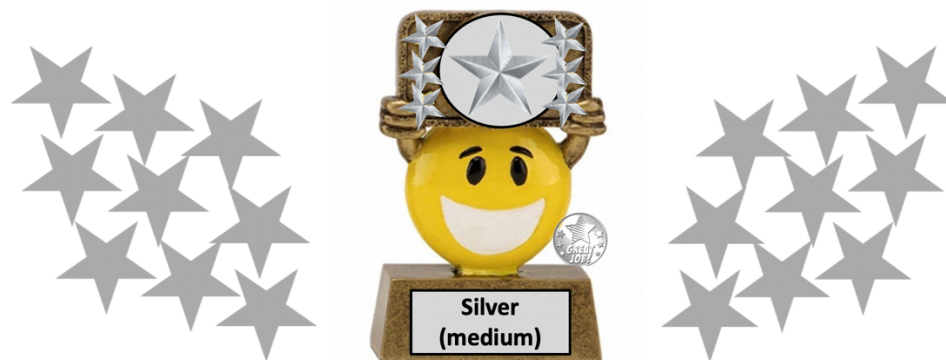
<p><b>Way 1: Use formula for sum of interior angles <math>180(n - 2)</math></b></p> <p>1 interior: <math>\frac{180(n-2)}{n} = 156</math></p> <p>Solve for <math>n</math>:</p> $\frac{180n - 360}{n} = 156$ $180n - 360 = 156n$ $24n = 360$ $n = \frac{360}{24} = 15$	<p><b>Way 2: Use formula for exterior angle <math>\frac{360}{n}</math></b></p> <p>Exterior angle = <math>180 - \text{interior angle}</math></p> <p>Interior angle = <math>180 - 156 = 24^\circ</math></p> <p>Exterior angle = <math>\frac{360}{24} = 15</math></p>
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14)

i.	
<p><b>Way 1: Use formula for sum of interior angles <math>180(n - 2)</math></b></p> <p>Interior angle = <math>180 - \text{exterior angle}</math></p> <p>Interior angle = <math>180 - 18 = 162^\circ</math></p> <p>1 interior: <math>\frac{180(n-2)}{n} = 150</math></p> <p>Solve for <math>n</math>:</p> $\frac{180n - 360}{n} = 162$	<p><b>Way 2: Use formula for exterior angle <math>\frac{360}{n}</math></b></p> <p>Exterior angle = <math>\frac{360}{18} = 20</math></p>

$180n - 360 = 162n$ $18n = 360$ $n = \frac{360}{18} = 20$	
<p>ii.</p> <p>Use formula for sum of interior angles <math>180(n - 2)</math></p> $\begin{aligned} \text{sum of all angles} &= 180(20 - 2) \\ &= 180(18) \\ &= 3240^\circ \end{aligned}$	

## 2 Silver



## 2.1 Working Out Angles

15)

Consider the Pentagon first	
<p><b>Way 1: Use formula for sum of interior angles <math>180(n - 2)</math></b></p> $\begin{aligned} \text{sum of all angles} &= 180(5 - 2) \\ &= 180(3) \\ &= 540 \end{aligned}$ <p>● 1 interior = <math>z = \frac{540}{5} = 108^\circ</math></p>	<p><b>Way 2: Use formula for exterior angle <math>\frac{360}{n}</math></b></p> $\text{Exterior angle} = \frac{360}{5} = 72^\circ$ <p>● Interior angle = <math>z = 180^\circ - \text{exterior}</math>  <math>= 180 - 72 = 108^\circ</math></p>
<p>Now consider the parallelogram. <b>Adjacent angles of a parallelogram add up to <math>180^\circ</math></b> (same side/co-interior angles)</p> <p>● <math>y = 180 - 117 = 63^\circ</math></p> <p>We know that entire interior angle is <math>108^\circ</math></p> <p>● <math>x = 108 - 63 = 45^\circ</math></p>	

16)

Consider the ABCDEF

i. Lines GH and AB are parallel  
 Way 1:  $x = 107^\circ$  since the angles are corresponding angles  
 Way 2.  $z = 180 - 107 = 73^\circ$  since same side/co-interior angles  
 $x = 180 - 73 = 107^\circ$  since straight lines angles add to  $180^\circ$

ii. Use formula for sum of interior angles  $180(n - 2)$

sum of all angles of GHCDEF =  $180(6 - 2)$   
 $= 180(4)$   
 $= 720$

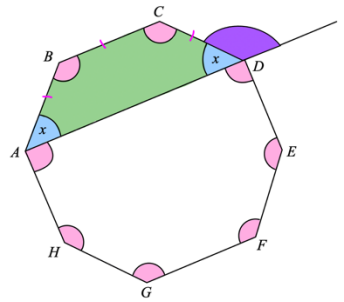
$y = 720 - 92 - 123 - 107 - 134 - 142 = 122^\circ$

17)

Consider the Pentagon

<p><b>Way 1: Use formula for sum of interior angles <math>180(n - 2)</math></b></p> <p><u>Pentagon:</u> sum of all angles = <math>180(5 - 2)</math>  <math>= 180(3)</math>  <math>= 540</math></p> <p><math>1 \text{ interior} = \angle CDE = \frac{540}{5} = 108^\circ</math></p> <p>Exterior angle = <math>\angle CDF = 180 - 108 = 72^\circ</math></p> <p>CDF is an isosceles triangle, therefore the base angles are equal</p> <p>The sum of the angles in a triangle is <math>180^\circ</math>.</p> <p><math>\angle CFD = 180 - 72 - 72 = 36^\circ</math></p>	<p><b>Way 2: Use formula for exterior angle <math>\frac{360}{n}</math></b></p> <p><u>Pentagon:</u></p> <p>Exterior angle = <math>\angle CDF = \frac{360}{5} = 72^\circ</math></p> <p>CDF is an isosceles triangle, therefore the base angles are equal</p> <p>The sum of the angles in a triangle is <math>180^\circ</math>.</p> <p><math>\angle CFD = 180 - 72 - 72 = 36^\circ</math></p>
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18)



Firstly, we consider the Octagon

Way 1: Use formula for sum of interior angles  $180(n - 2)$

$$\begin{aligned} \text{sum of all angles} &= 180(8 - 2) \\ &= 180(6) \\ &= 1080 \end{aligned}$$

● 1 interior =  $\frac{1080}{8} = 135^\circ$

Way 2: Use formula for exterior angle  $\frac{360}{n}$

$$\text{Exterior angle} = \frac{360}{8} = 45^\circ$$

● Interior angle =  $180^\circ - \text{exterior}$   
 $= 180 - 45 = 135^\circ$

Next, we consider the green quadrilateral

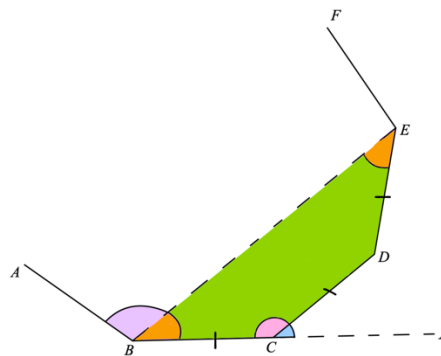
ABCD is a quadrilateral. Therefore, the sum of angles is  $360^\circ$  and the base angles  $x$  are equal since ABCD is an isosceles trapezoid

●  $x = \frac{360 - 135 - 135}{2} = 45^\circ$

Angles on a straight line add to  $180^\circ$

$$\angle CDJ = 180 - 45 = 135^\circ$$

19)



Firstly, we consider the nonagon

i. Use formula for exterior angle  $\frac{360}{n}$

● 1 exterior =  $\frac{360}{9} = 40^\circ$

angle DCX =  $40^\circ$

ii. Use formula for interior angle  
 Interior angle =  $180^\circ - \text{exterior}$

● Interior angle =  $180^\circ - \text{exterior}$   
 $= 180 - 40 = 140^\circ$

angle BCD =  $140^\circ$

iii. Now we look at BCDE which is a quadrilateral. Therefore, sum of angles is  $360^\circ$  and base angles are equal.

$$\bullet \angle CBE = \angle BED = \frac{360 - 140 - 140}{2} = 40^\circ$$

$$\bullet \text{Angle ABE} = 140 - 40 = 100^\circ$$

3 Gold



3.1 Working Out Angles

20)

Consider the Octagon

- $\angle BOC = \frac{360}{8} = 45^\circ$
- $\angle OBC = \frac{180-45}{2} = 67.5^\circ$  (isosceles triangle)

$\angle AOD = 3(45) = 135^\circ$

- $\angle OAD = \frac{180-135}{2} = 22.5^\circ$  (isosceles triangle)

21)

Consider the ABCDEF

$\angle BJK = 360 - 140 = 220^\circ$  (angles at a point add to  $360^\circ$ )

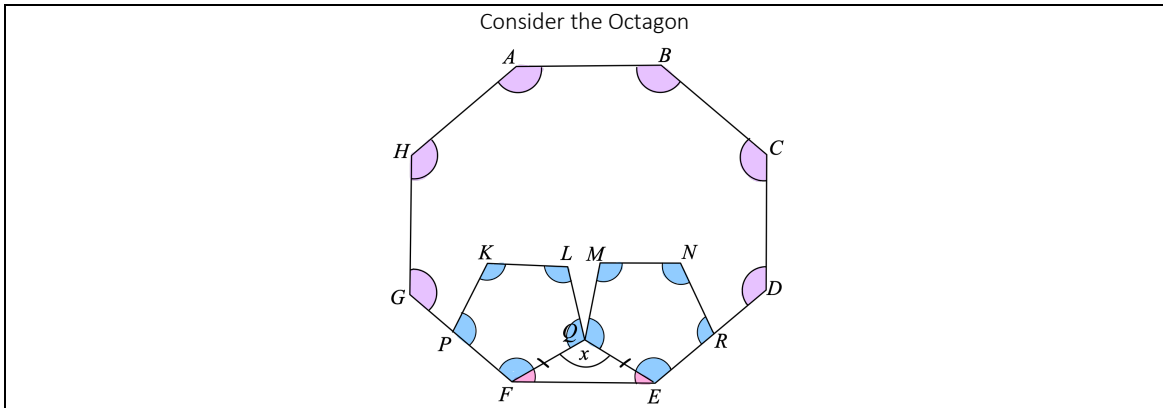
- $\frac{180(8-2)}{8} = 135^\circ$  (interior angles of a hexagon)

ABJGH is a pentagon therefore,

Sum of interior angles =  $180(5 - 2) = 540^\circ$

●  $\frac{540 - 220 - 135 - 135}{2} = 25^\circ$

22)



Way 1: Use formula for sum of interior angles  $180(n - 2)$

Octagon: sum of all angles =  $180(8 - 2)$   
 $= 180(6)$   
 $= 1080$

● 1 interior =  $\frac{1080}{8} = 135^\circ$

Pentagon: sum of all angles =  $180(5 - 2)$   
 $= 180(3)$   
 $= 540$

● 1 interior =  $\frac{540}{5} = 108^\circ$

Way 2: Use formula for exterior angle  $\frac{360}{n}$

Octagon: Exterior angle =  $\frac{360}{8} = 45^\circ$

● Interior angle =  $180^\circ - \text{exterior}$   
 $= 180 - 45 = 135^\circ$

Pentagon: Exterior angle =  $\frac{360}{5} = 72^\circ$

● Interior angle =  $180^\circ - \text{exterior}$   
 $= 180 - 72 = 108^\circ$

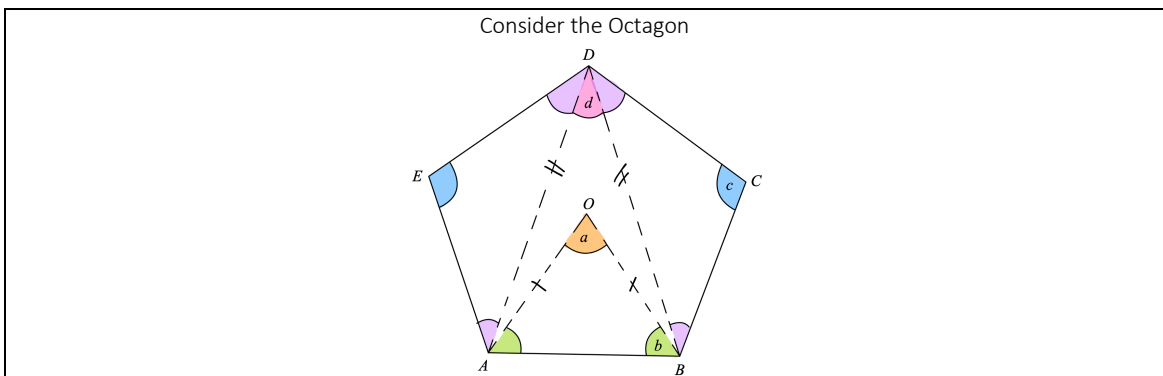
QEF is an isosceles triangle. Therefore, sum of angles is  $180^\circ$  and the base angles are equal

●  $\angle QFE = \angle QEF = 135 - 108 = 27$

z

$\angle EQF = x = 180 - 27 - 27 = 126^\circ$

23)





Use formula for exterior angle  $\frac{360}{n}$

$$a = \frac{360}{5} = 72^\circ$$

$$b = \frac{180 - 72}{2} = 54^\circ$$

$$c = \frac{180(5 - 2)}{5} = 108^\circ$$

$$e = \frac{180 - 108}{2} = 36^\circ$$

$$d = 108 - 36 - 36 = 36^\circ$$

Note: We could have also looked at triangle ADB for find  $d$

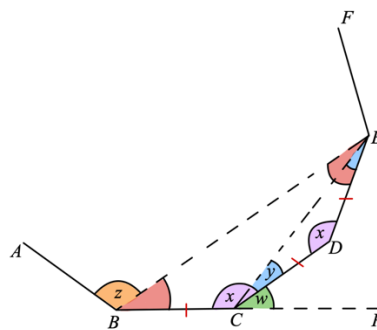
Angle OBD=angle DAO=  $108 - 54 - 36 = 18$  (we know the full exterior angle is 108)

Reflex angle AOB=  $360 - 72 = 288$  (angles at a point add to  $360^\circ$ )

$$360 - 288 - 18 - 18 = 36^\circ$$

24)

Consider the Decagon



$$x = \frac{180(10-2)}{10} = 144^\circ \text{ (interior angles of a decagon)}$$

$$y = \frac{180-144}{2} = 18^\circ \text{ (isosceles triangle)}$$

$$\text{exterior angle of decagon} = \frac{360}{n} = \frac{360}{10} = 36^\circ$$

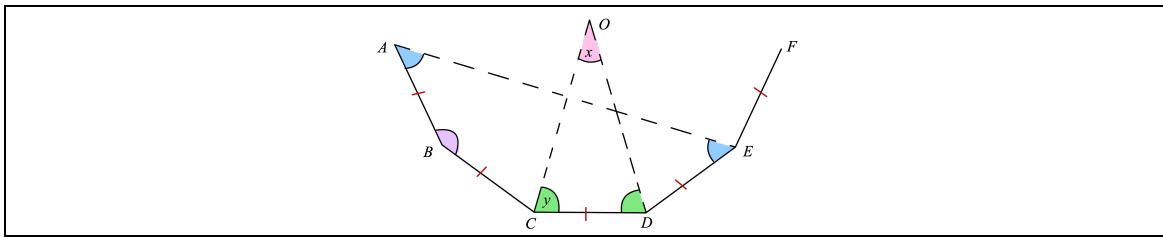
$$w + y = 36$$

$$w = 36 - y = 36 - 18 = 18$$

BCDE is an isosceles trapezium  $\angle CBE = \angle BED = \frac{360 - 144 - 144}{2} = 36^\circ$

$$z = 144 - 36 = 108^\circ$$

25)



i.  $x : y = 1 : 2$

This means:

$$\frac{x}{y} = \frac{1}{2}$$

Re – arranging gives:

●  $y = 2x$

We know the sum of the angles of a triangle is  $180^\circ$  so we can form an equation:

$$2x + 2x + x = 180$$

$$5x = 180$$

●  $x = 36^\circ$

ii. The angle at the centre  $x = \frac{360}{n}$   
where  $n$  = number of sides

$$36 = \frac{360}{n}$$

Solving for  $n$  gives

$$n = 10$$

We can plug this in the formula for an interior angle

$$\frac{180(10 - 2)}{10} = 144^\circ$$

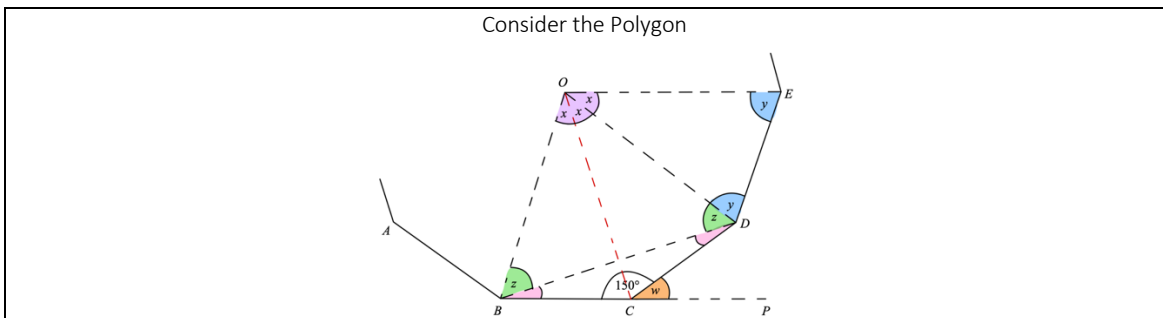
iii. ABCDE is a 5-sided shape

$$180(5 - 2) = 540^\circ \text{ (sum of interior angles)}$$

The base angles of this 5-sided shape are equal

●  $\frac{540 - 3(144)}{2} = 54^\circ$

26)



i.  $w = 180 - 150 = 30^\circ$  (angles on a straight line)

ii.  $\frac{360}{30} = 12$

iii.

●  $x = \frac{360}{12} = 30^\circ$

ODE is an isosceles triangle, so base angle are equal

$$y = \frac{180 - 30}{2} = 75^\circ$$

●  $\frac{180-150}{2} = 15$

●  $z = \frac{180-30-30}{2} = 60^\circ$  using isosceles triangle OBD  
or

$z = 150 - 75 - 15 = 60^\circ$  using angle D = 150°

iv. Equilateral

27)

Consider the Polygon

i. ●  $e = \frac{360}{15} = 24^\circ$

ii.  $180 - 24 = 156^\circ$

iii. ABCD is a quadrilateral therefore, the sum of all angles is 360  
●  $x = \frac{360-156-156}{2} = 24^\circ$

iv. ●  $y = 156 - 24 - 24 = 108^\circ$

### 3.2 Working Out The Number Of Sides

28)

Consider the Pentagon

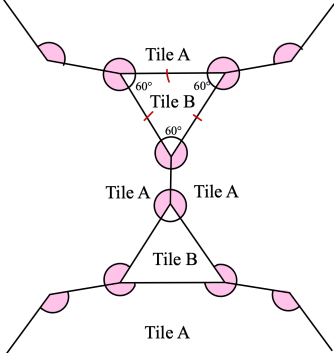
<p>Way 1: Use formula for sum of interior angles <math>180(n - 2)</math></p> <p>sum of all angles = <math>180(5 - 2)</math> = <math>180(3)</math> = 540</p> <p>● 1 interior = <math>x = \frac{540}{5} = 108^\circ</math></p>	<p>Way 2: Use formula for exterior angle <math>\frac{360}{n}</math></p> <p>Exterior angle = <math>\frac{360}{5} = 72^\circ</math></p> <p>● Interior angle = <math>x = 180^\circ - \text{exterior} = 180 - 72 = 108^\circ</math></p>
<p>We have two pentagons connected to polygon A at point P. We also know that angles at a point add to <math>360^\circ</math> and</p> <p>● Interior angle of A = <math>n = 360 - 108 - 108 = 144^\circ</math></p>	

$$\text{Exterior angle of A} = 180 - 144 = 36^\circ$$

$$\text{Therefore, } n = \frac{360}{36} = 10$$

29)

Consider the Polygon



Tile B (triangle) must be equilateral since it is a regular triangle.

$$360 - 60 = 300^\circ$$

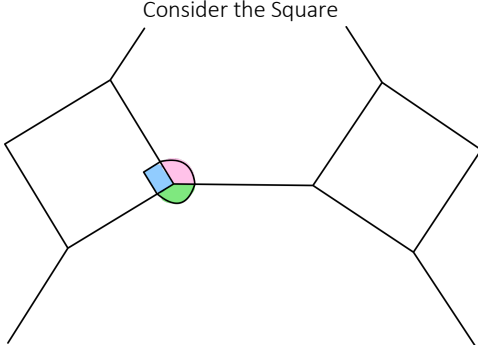
● 1 interior =  $\frac{300}{2} = 150^\circ$

1 exterior  $180 - 150 = 30^\circ$

$$n = \frac{360}{\text{exterior angle}} = \frac{360}{30} = 12$$

30)

Consider the Square



We have a square so the we have a  $90^\circ$  angle.

● Interior angles of hexagon =  $\frac{180(6-2)}{6} = 120^\circ$

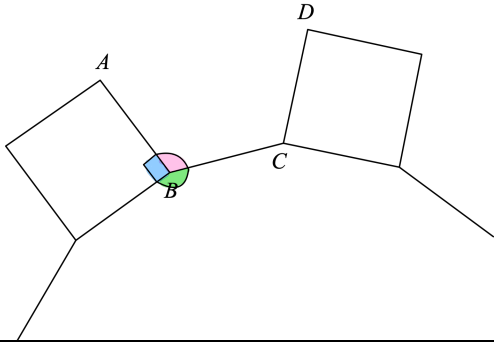
● 1 interior of n sided shape =  $360 - 120 - 90 = 150^\circ$

1 exterior  $180 - 150 = 30^\circ$

$$n = \frac{360}{\text{exterior angle}} = \frac{360}{30} = 12$$

31)

Consider the Square



We have a square so the we have a perpendicular angle.

- Interior of 12-sided polygon =  $\frac{180(12-2)}{12} = 150^\circ$
- 1 interior =  $360 - 150 - 90 = 120^\circ$

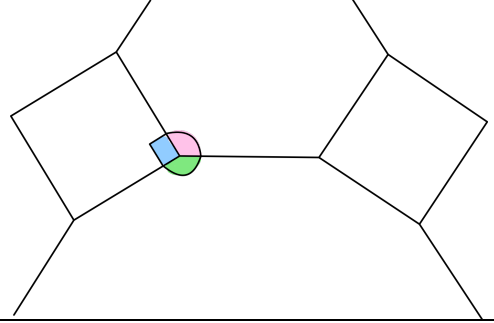
1 exterior of P =  $180 - 120 = 60^\circ$

$$n = \frac{360}{\text{exterior angle}} = \frac{360}{60} = 6$$

therefore, we have a hexagon.

32)

Consider the Square



We have a square so the we have a perpendicular angle.

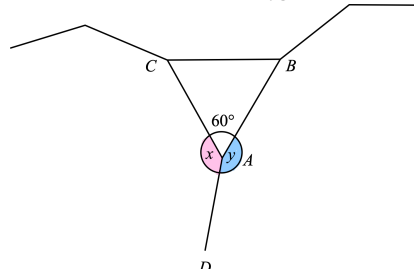
- Interior of hexagon =  $\frac{180(6-2)}{6} = 120^\circ$
- 1 interior =  $360 - 120 - 90 = 150^\circ$

1 exterior  $180 - 150 = 30^\circ$

$$n = \frac{360}{\text{exterior angle}} = \frac{360}{30} = 12$$

33)

Consider the Polygon



●  $y = \frac{180(10 - 2)}{10} = 144^\circ$

●  $x = 360 - 144 - 60 = 156^\circ$

1 exterior  $180 - 156 = 24^\circ$

$n = \frac{360}{\text{exterior angle}} = \frac{360}{24} = 15$

34)

Angles at a point add up to  $360^\circ$

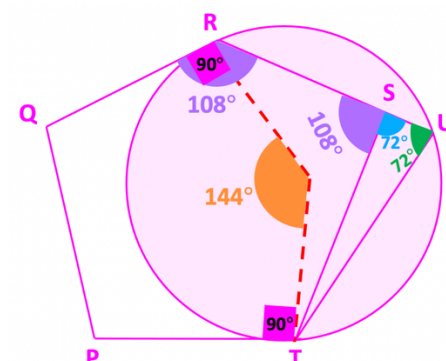
remaining angle at the centre =  $360 - 108 - 90 = 162^\circ$

Exterior angle of the shape =  $180 - 162 = 18^\circ$

number of sides =  $\frac{360}{\text{exterior angle}} = \frac{360}{18} = 20$  sides

sum of interior angles =  $180(20 - 2) = 3240^\circ$

35)



Interior angle of a regular pentagon  $\frac{180(5-2)}{5} = 108^\circ$

Angle TSU =  $180 - 108 = 72^\circ$  (angles on a straight line add to  $180^\circ$ )

Form the 2 lines OR and OT

Angle QRO = Angle PTO =  $90^\circ$  (a tangent meets a radius at  $90^\circ$ )

Angle ROT =  $540 - (90 + 90 + 108 + 108) = 144^\circ$  (angles in Pentagon PQRST add to  $540^\circ$ )

Angle RUT =  $\frac{144}{2} = 72^\circ$  (angle at the centre is twice the angle at the circumference)

So, we have angle RUT =  $72^\circ$  and that Angle RUT =  $72^\circ$

Hence the base angles of triangle SUT are equal, so SUT is an isosceles triangle  $\Rightarrow ST = UT$

## 4 Diamond



36)

Consider the ABCDEF

Let angle CDE be  $x$ .  
This means that BCD is:

$$\begin{aligned} \text{Angle BCD} &= 2 (\text{angle CDE}) \\ &= 2x \end{aligned}$$

These are added to the diagram along with the [line of symmetry](#). And due to the [line of symmetry](#),

angle A = angle B  
angle F = angle C  
angle E = angle D

Use formula for sum of interior angles  $180(n - 2)$

$$\begin{aligned} 180(6 - 2) &= 720 \\ 117 + 117 + 2x + 2x + x + x &= 720 \\ 234 + 6x &= 720 \\ 6x &= 486 \\ x &= 81^\circ \end{aligned}$$

●  $\angle AFE = 2x = 2(81) = 162^\circ$

37)

Consider the Polygon

i. $4v + v = 180^\circ$ $5v = 180^\circ$ $v = 36^\circ$	ii. $\frac{360}{36} = 10$
iii. ● $\frac{180-4(36)}{2} = 18^\circ$	iv. ● $\frac{180(10-2)}{10} = 144^\circ$ Or $4v = 4(36) = 144^\circ$  ● $144 - 18 = 126^\circ$
v. CDEF is a quadrilateral (isosceles trapezoid with equal base angles)  ● $\frac{360-144-144}{2} = 36^\circ$	

38)

<p><b>Way 1: Use formula for an interior angle <math>\frac{180(n-2)}{n}</math> and an exterior angle is <math>\frac{360}{n}</math></b></p> <p style="text-align: center;">Exterior angle <math>\frac{360}{n}</math>                  Interior angle <math>180 - \frac{360}{n}</math>  <math>180 - \frac{360}{n} = 6.5 \left( \frac{360}{n} \right)</math>  <math>180 - \frac{360}{n} = \frac{2340}{n}</math>  <math>180 = \frac{2700}{n}</math>  <math>180n = 2700</math>  <math>n = \frac{270}{180} = 15</math> sides</p> <p>Yes since <math>n</math> is a whole number. Each interior angle is <math>156^\circ</math> and each exterior angle is <math>24^\circ</math></p>	<p><b>Way 2: Use fact that interior angle + exterior angle adds to <math>180^\circ</math></b></p> <p style="text-align: center;">Call an exterior angle <math>x</math>                  An interior angle is <math>6.5x</math></p> <p>We know these angles add to <math>180^\circ</math> since they lie on a straight line</p> <p style="text-align: center;"><math>x + 6.5x = 180</math>  <math>7.5x = 180</math>  <math>x = 24^\circ</math></p> <p>Yes. Each interior angle is <math>156^\circ</math> and each exterior angle is <math>24^\circ</math>. All interior angles are the same and all exterior angles are the same, therefore the shape is regular</p>
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39)

The sum of the interior angles of a hexagon  $180(6 - 2) = 720^\circ$

Sum of angles given =  $79 + 42 + 49 + 52 + 97 = 319^\circ$

$720 - 319 = 401^\circ$

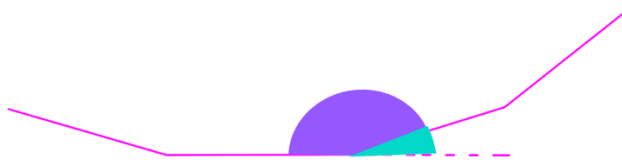
So the 6<sup>th</sup> angle is  $401^\circ$

Interior + exterior =  $180^\circ$  so it is not possible for either an interior or exterior angle to be  $180^\circ$  or more.

There  $401^\circ$  is not a possible answer for an interior angle, hence the shape cannot be a hexagon



40) .



<p>Way 1: Use formula for an interior angle <math>\frac{180(n-2)}{n}</math> and an exterior angle is <math>\frac{360}{n}</math></p> $\frac{180(n-2)}{n} = 140 + \frac{360}{n}$ $\frac{180n-360}{n} = 140 + \frac{360}{n}$ <p>Multiply all terms by <math>n</math></p> $180n - 360 = 140n + 360$ $180n - 140n = 360 + 360$ $40n = 720$ $n = 18$	<p>Way 2: Use fact that interior angle + exterior angle adds to <math>180^\circ</math></p> <p>Call  an interior angle <math>x</math>  an exterior angle <math>y</math></p> <p>We can build 2 equations</p> <p>①: <math>x + y = 180</math>  ②: <math>x = y + 140</math></p> <p>Solve simultaneously</p> $y + 140 + y = 180$ $2y + 140 = 180$ $2y = 40$ $y = 20$ <p>So we have an exterior angle is 20</p> <p>We know the formula for an exterior angle is <math>\frac{360}{n}</math></p> $\frac{360}{20} = 18$
<p>Way 3: Use formula for an interior angle <math>\frac{180(n-2)}{n}</math> and an exterior angle is <math>180 - \text{interior angle}</math></p> $\frac{180(n-2)}{n} = 140 + \left(180 - \frac{180(n-2)}{n}\right)$ $\frac{180n-360}{n} = 140 + \left(180 - \frac{180n-360}{n}\right)$ <p>Multiply all terms by <math>n</math></p> $180n - 360 = 140n + 180n - (180n - 360)$ $180n - 360 = 140n + 180n - 180n + 360$ $180n - 140n - 140n = 360 + 360$ $40n = 720$ $n = 18$	

41)

$7x + x = 180^\circ$ $8x = 180^\circ$

$$\bullet x = 22.5^\circ$$

$$n = \frac{360}{\text{exterior angle}}$$

$$n = \frac{360}{22.5} = 16$$

42)

$$11x + x = 180^\circ$$

$$12x = 180^\circ$$

$$x = 15^\circ$$

$$n = \frac{360}{\text{exterior angle}}$$

$$n = \frac{360}{15} = 24$$